Section 2: Mathematics

Mathematics Question 1

Choice (D) is correct. Since $x = 1.38$, it follows that $2x = 2.76$. Rounded to the nearest tenth, 2.76 is 2.8.

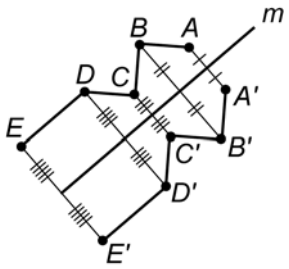
Mathematics Question 2

Choice (A) is correct. The cost of 6 calendars is $6k$ dollars, and the cost of 3 notebooks is $3p$ dollars, so the total cost of 6 calendars and 3 notebooks is $6k + 3p$ dollars. Since all of the answer choices are in factored form, it would be helpful to factor $6k + 3p$ to determine the correct answer among the answer choices. Factoring this algebraic expression gives $3(2k + p)$.

Mathematics Question 3

Choice (A) is correct. The diagram contains a line, labeled m, and, to the left of m, a figure made up of four line segments connected end to end. Imagine folding the page so that line m is the fold line. The set of connected line segments would coincide with a set of line segments on the other side of line m, the image of the original set of segments.

Each point on the original figure must correspond to a point on the other side of m that is the same distance from m. For example, in the figure below, A corresponds to A', where A' is the same distance from m as A is. B corresponds to B', and so forth.



When you connect A', B', C', D', and E', the result is the reflection of the original figure.

You could also look at the answer choices to help you determine the answer to the question. In choice (C), the figure is not the result of reflecting the set of line segments about line m; it is the result of sliding the set of line segments across line m. In choice (D), the segments have been rearranged. In choices (B) and (E), two of the angles have been moved from upper right to lower left.

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Mathematics Question 4

Choice (B) is correct. Using the first equation, $x = 2 + y$, you can substitute $2 + y$ for x in the second equation. When you make this substitution, you get $3y = 2 + y$. This equation simplifies to $2y = 2$, which means that $y = 1$.

Mathematics Question 5

Choice (C) is correct. The line through points A and B is perpendicular to the y-axis, so it must be parallel to the x-axis. Therefore, every point on the line has the same y-coordinate. Since the y-coordinate of A is 2, the y-coordinate of B is also 2. Since $AB = 7$ and the x-coordinate of A is -4, it follows that the x-coordinate of B is $-4 + 7$, which equals 3. So the coordinates of B are $(3, 2)$.

You could also answer this question by eliminating answer choices that are not reasonable. For example, the coordinates in (B) and (D) are coordinates of points on the x-axis, not on line segment $\overline{AB}$. The coordinates in (A) are coordinates of a point below the x-axis, because the y-coordinate is negative. The only two choices left to consider are (C) and (E). Choice (E) can be eliminated since $(7, 2)$ are the coordinates of a point that is farther from the y-axis than point A. Therefore, (E) is not a reasonable answer to the question.

Mathematics Question 6

Choice (C) is correct. One approach to this question is to try out an allowable value of n, an integer greater than 2, and see which answer choices are greater than 1. If $n = 3$, the answer choices have the following values: (A) $\frac{3}{4}$, (B) $\frac{3}{7}$, (C) $\frac{6}{4}$, (D) $\frac{6}{10}$, and (E) $\frac{9}{10}$. The only one of these that is greater than 1 is choice (C), since $\frac{6}{4} > 1$.

You may also be able to eliminate answer choices without substituting numbers into each expression. For example, the expressions in (A), (B), and (E) cannot be greater than 1, because the numerator in each of these expressions is less than the denominator and both the numerator and the denominator are positive. At this point, you could substitute $n = 3$ into the expressions in (C) and (D) to eliminate choice (D).

Mathematics Question 7

Choice (E) is correct. Since July 1 is a Monday, it follows that July 8, July 15, July 22, and July 29 are also Mondays (adding multiples of 7 days to July 1). Therefore, July 30 and 31 and August 1, 2, and 3 are the following days: Tuesday, Wednesday, Thursday, Friday, Saturday. So August 3 is a Saturday.

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Mathematics Question 8

Choice (A) is correct. Use the definition of $a \text{ n } b$ to evaluate $(8 \text{ n } 8) - (6 \text{ n } 6)$, which gives how much larger $8 \text{ n } 8$ is than $6 \text{ n } 6$. Since $8 \text{ n } 8 = \frac{8 \times 8}{8 + 8} = \frac{64}{16} = 4$ and $6 \text{ n } 6 = \frac{6 \times 6}{6 + 6} = \frac{36}{12} = 3$, it follows that $(8 \text{ n } 8) - (6 \text{ n } 6) = 4 - 3 = 1$.

Mathematics Question 9

Choice (C) is correct. Any month in which Monica sold a greater number of cars than she had sold in the previous month will appear on the graph as a point that is higher than the point for the previous month. The months that satisfy this condition are March, May, and June, so there are three months in which Monica sold a greater number of cars than she sold in the previous month.

Mathematics Question 10

Choice (E) is correct. When division is carried out within the set of integers, the result is a quotient and a remainder. Consider the following division results:

$16 \div 5 = 3$ remainder 1,
$17 \div 5 = 3$ remainder 2,
$18 \div 5 = 3$ remainder 3,
$19 \div 5 = 3$ remainder 4,
$20 \div 5 = 4$ remainder 0.
$21 \div 5 = 4$ remainder 1,
and so on.

When you divide by 5, the remainder can be 0, 1, 2, 3, or 4, but not 5, because the remainder has to be less than the divisor.

Mathematics Question 11

Choice (C) is correct. Right triangle ABC has $AB = 3$ and $BC = 4$. The area of this triangle is $\frac{1}{2} \times 3 \times 4 = 6$. By the Pythagorean theorem, the length of the hypotenuse, $\overline{AC}$, is 5 since $AC^2 = 3^2 + 4^2 = 9 + 16 = 25$. Since $AC = 5$, the length of each side of the square is 5, and the area of the square is 5^2, or 25. The total area of $ABCDE$ is thus $6 + 25 = 31$.

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Mathematics Question 12

Choice (E) is correct. Translating the information about x gives the equation $\sqrt{x} - 3 = 8$. From this, it follows that $\sqrt{x} = 11$, and therefore, $x = 11^2 = 121$. The value of $x - 1$ is $121 - 1 = 120$.

Mathematics Question 13

Choice (D) is correct. The total number of homes in the survey was $2 + 9 + 18 + 4 + 2 + 1 = 36$. There were 9 homes having exactly 1 telephone each. The fraction $\frac{9}{36}$ can be simplified to $\frac{1}{4}$.

Mathematics Question 14

Choice (B) is correct. The comparison of 0.123 with $\frac{n}{100}$ and $\frac{n+1}{100}$ is easier if all three numbers are in the same form, so rewriting 0.123 as $\frac{12.3}{100}$ will help you see the comparison more clearly. Since $\frac{n}{100} < \frac{12.3}{100} < \frac{n+1}{100}$, it follows that 12.3 is between n and $n + 1$. The only integer n that satisfies this condition is 12, because $12 < 12.3 < 12 + 1$.

Mathematics Question 15

Choice (E) is correct. You are given that the average of c, d, and e is greater than the average of d, e, and f, which means that the sum of c, d, and e is greater than the sum of d, e, and f. The only way that this can be true is if c is greater than f. This can be stated algebraically: since $\frac{c+d+e}{3} > \frac{d+e+f}{3}$, you can multiply both sides of the inequality by 3 and get $c + d + e > d + e + f$, and then you can subtract $d + e$ from both sides and get $c > f$.

Similarly, since the average of d, e, and f is greater than the average of e, f, and c, you know that d is greater than c, and since the average of e, f, and c is greater than the average of f, c, and d, you know that e is greater than d. Once you know that $c > f$, $d > c$, and $e > d$, it follows that $e > d > c > f$.

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Mathematics Question 16

Choice (E) is correct. An important theorem about triangles is the triangle inequality theorem; this theorem states that in any triangle, the sum of the lengths of any two of the sides must be greater than the length of the third side. For example, 2, 3, and 1 cannot be the lengths of the sides of a triangle, since $2 + 1$ is not greater than 3. You can try to construct a triangle with sides of lengths 2, 3, and 1, and you will see that it collapses into a line segment. The same thing happens if you try to construct a triangle with sides of lengths 3, 6, and 9. The lengths collapse into a line segment. The numbers 3, 10, and 6 cannot be the lengths of the sides of a triangle, according to the triangle inequality, since $3 + 6 < 10$. Likewise, 4, 7, and 2 cannot be the lengths of the sides of a triangle since $4 + 2 < 7$.

Among the answer choices, the only set of numbers that obeys the triangle inequality is the set 7, 8, and 9.

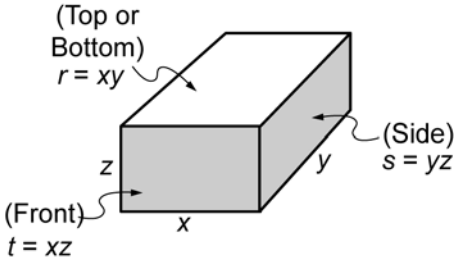
Mathematics Question 17

Choice (D) is correct. In the *xy*-plane, if you have two lines that are not parallel to either the *x*-axis or the *y*-axis and the two lines are perpendicular, then their slopes must be negative reciprocals of each other. That is, the product of the slopes must be -1. The equation $2x + y = 3$ can be rewritten as $y = -2x + 3$, and from this you can see that the slope of this line is -2. The slope of the other line is *m*. Since *m* is the negative reciprocal of -2, it equals $\frac{1}{2}$. (Note that $-2 \times \frac{1}{2} = -1$.)

Mathematics Question 18

Choice (D) is correct. In this question, you are not given the dimensions of the box, so let the dimensions of the rectangular box be *x* by *y* by *z.* The volume of the box can then be expressed as *xyz.*

You are given that the areas of the bottom, side, and front of the box are *r*, *s*, and *t*. This means that $r = xy$, $s = yz$, and $t = xz$, as shown below.



From these equations, it follows that $rst = (xy)(yz)(xz) = x^2y^2z^2 = (xyz)^2$. Since the volume of the box is *xyz*, solving the equation $rst = (xyz)^2$ for *xyz* yields $\sqrt{rst} = xyz$.

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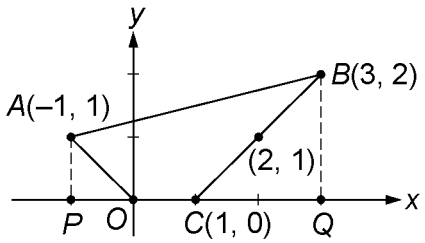
Mathematics Question 19

Choice (B) is correct. The train traveled for 1 minute and 10 seconds (which is 70 seconds) at 20 meters per second. Using the formula $\text{rate} \times \text{time} = \text{distance}$, you can see that the train traveled $20 \times 70 = 1{,}400$ meters. This period of traveling began when the front of the train entered the tunnel and ended when the back of the train left the tunnel, at which point the front of the train was 200 feet beyond the tunnel (since 200 meters is the length of the train). Thus, the train traveled the length of the tunnel plus 200 meters. Therefore, $(\text{length of tunnel}) + 200 = 1{,}400$. It follows that the length of the tunnel is 1,200 meters.

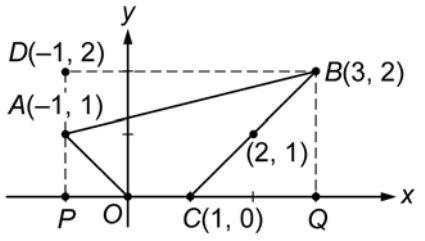
Mathematics Question 20

Choice (B) is correct. One way to approach this problem is to drop a perpendicular from *A* to the *x*-axis, intersecting the *x*-axis at point *P,* whose coordinates are $(-1, 0)$, and to drop a perpendicular from *B* to the *x*-axis, intersecting the *x*-axis at point *Q*, whose coordinates are $(3, 0)$. The resulting figure is



Then find the area of *PABQ* and from that subtract the area of n*PAO* and the area of n*CBQ*. Since *PABQ* is a trapezoid with parallel bases $\overline{AP}$ and $\overline{BQ}$ and altitude $\overline{PQ}$, its area is $\frac{1}{2}(1+2) \times 4$, which equals 6. The area of n*PAO* is $\frac{1}{2} \times 1 \times 1$, which equals $\frac{1}{2}$. The area of n*CBQ* is $\frac{1}{2} \times 2 \times 2$, which equals 2. Therefore, the area of *OABC* is $6 - \left(\frac{1}{2} + 2\right)$, which is $3\frac{1}{2}$.

Alternatively, you could have inserted additional lines in the figure to form a rectangle. Using the resulting figure shown below, you could find the area of *OABC* by subtracting the areas of <u>three</u> triangles (n*PAO*, n*ADB*, and n*CBQ*) from the area of rectangle *PDBQ*.



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The areas of $\triangle PAO$ and $\triangle CBQ$ were found in the solution above to be $\frac{1}{2}$ and 2, respectively. The area of $\triangle ADB$ is $\frac{1}{2} \times 1 \times 4 = 2$. The area of rectangle $PDBQ$ is 4×2, or 8. Therefore, the area of $OABC$ is $8 - \frac{1}{2} - 2 - 2$, which equals $3\frac{1}{2}$.

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